Phase Analysis of Sweeping Probe Data using the Hilbert Transform

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Purpose & Outline

(a) **Purpose**

1. Describe application of Hilbert transform for analyzing sweeping probe data
2. Highlight when HT is useful, along with its restrictions

(b) **Outline**

1. Background on Hilbert transform
2. Implementation in practice
3. Example with measuring spoke oscillations
4. Conclude

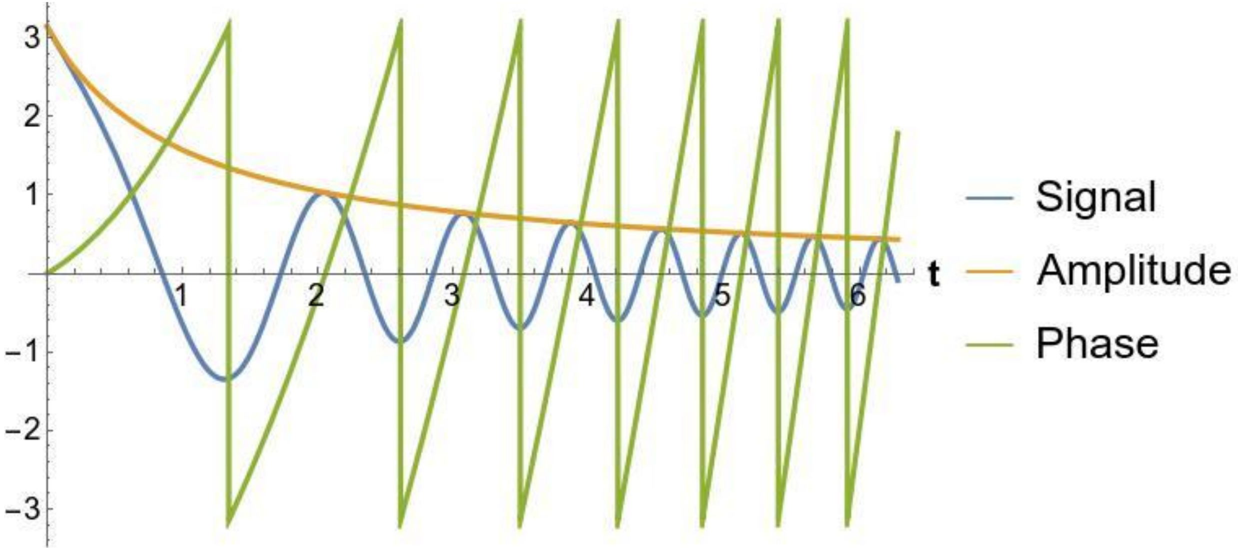
Background : The Hilbert transform (1/2)

- The Hilbert transform $\mathrm{H}[n(t)]$ of a real time series $n(t)$ is chosen such that when

$$\mathrm{S}_n(t) = n(t) + i\mathrm{H}[n(t)] = \mathrm{A}_n(t)e^{i\theta_n(t)}$$

is analytically continued into the complex plane, $\mathrm{A}_n(t)$ corresponds to the slow varying envelope and $\theta_n(t)$ to the fast varying instantaneous phase

- Advantage over Fourier transform is that it can track a non-stationary signal



Background : The Hilbert transform (2/2)

- For example, one would want a pure cosine mode to transform into:

$$cos(\omega t) + i\mathrm{H}[cos(\omega t)] = e^{i\omega t}$$

- This motivates rotating every Fourier component by $-\pi/2$. i.e.

$$F(\mathrm{H}(n))(\omega) = -i\,\mathrm{sgn}(\omega)F(n)(\omega)$$

- Which, by the convolution theorem, gives the formal Hilbert transform:

$$\mathrm{H}(n(t)) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{n(\tau)}{t-\tau}d\tau$$

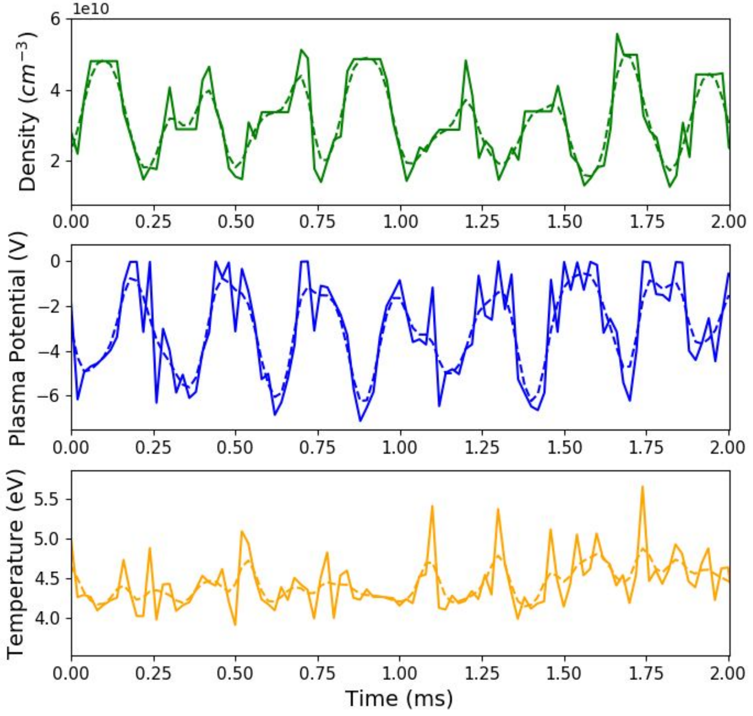
Implementation in Practice

Given several discrete time series $\{n_1(t), n_2(t), ...\}$, multiple probes or different plasma parameters:

1. Bandpass filter each around the oscillatory mode of interest
   (a) Meaningful HT requires a dominant mode
2. Apply HT to each
   (a) $n(t) \rightarrow \mathrm{FFT} \rightarrow \text{Multiplication by } -i\,\mathrm{sgn}(\omega) \rightarrow \mathrm{iFFT} = \mathrm{H}(n(t))$
3. Extract $\mathrm{A}(t)$, $\theta(t)$ from each
   (a) $\mathrm{A}(t) = \sqrt{n(t)^2 + \mathrm{H}(n(t))^2}$
   (b) $\theta(t) = \arctan(\frac{\mathrm{H}(n(t))}{n(t)})$
4. Analysis, several options:
   (a) Study time dependent phase differences (e.g. $\theta_2(t) - \theta_1(t)$ between multiple probes or different plasma parameters)
   (b) Study relative amplitude growths
   (c) **Make a phase plot; bin all time series by phase of chosen reference time series**

Application to Sweeping Probe Data (1/2)

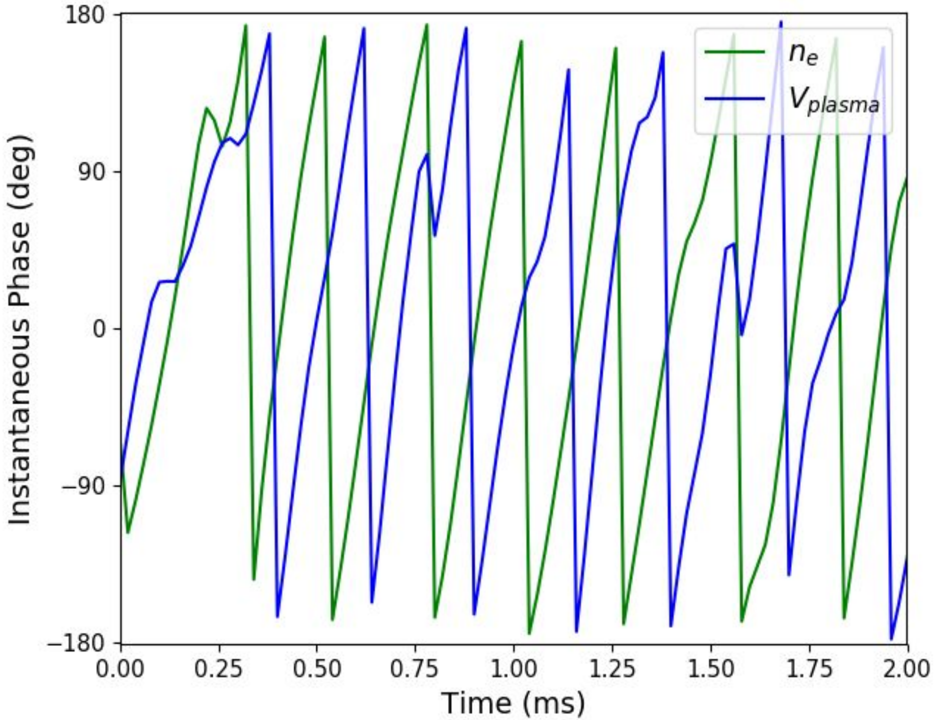
Fast Sweeping Probe Time Series Inside a Spoke



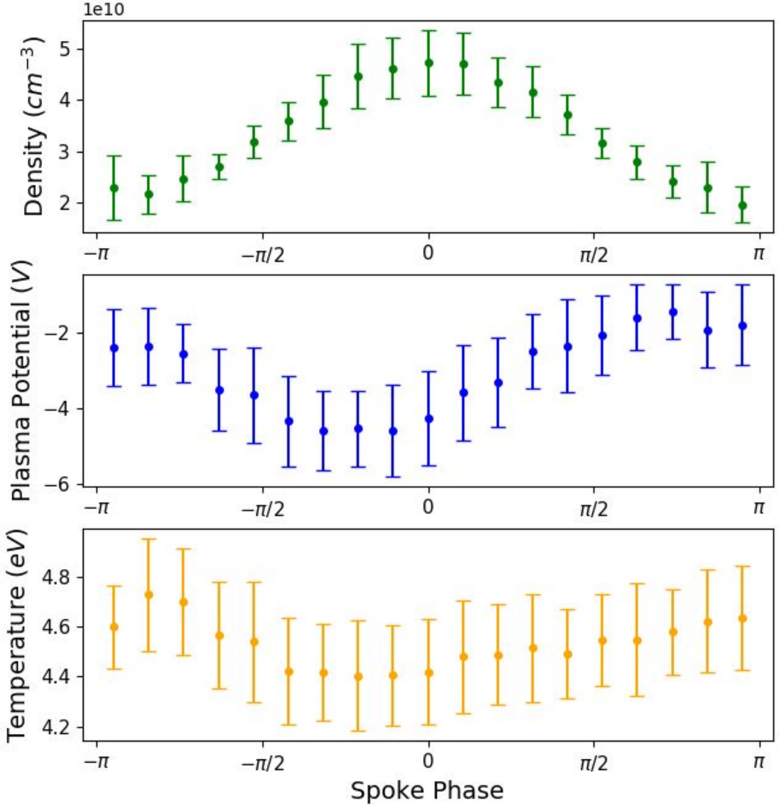
Apply HT

Extract θ(t)

Instantaneous Phases



Application to Sweeping Probe Data (2/2)



1. Phase plot (left) result of binning by the instantaneous phase of the density
2. Using azimuthal variation of potential, can estimate cross-field current

$$I_a = \langle env_{E\times B} \rangle 2\pi RL = -\frac{e\pi L}{B}\delta n \delta V sin(\theta_{\delta n,\delta V})$$

3. Using phase plot data $I_a = 0.4$A, which is 33% of the discharge current (1.2A) during this experiment.

Summary & Conclusion

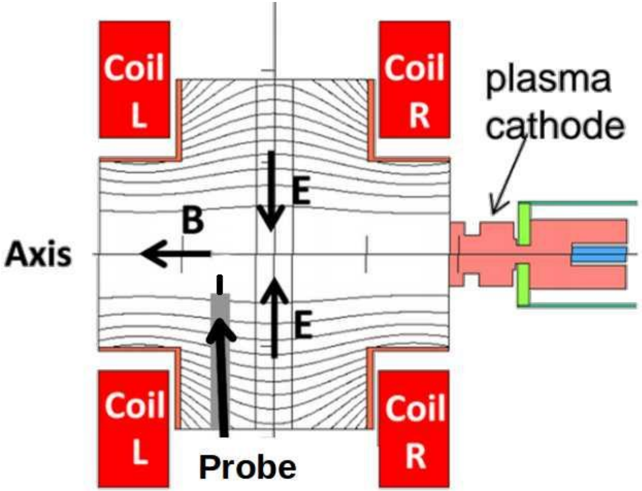
(a) HT can be used for non-stationary signal analysis

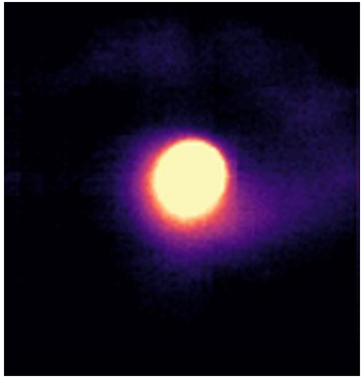
1. Measure time variation of the amplitude and phase of a non-stationary signal
2. Analyze instantaneous phase differences between signals (between multiple probes, different plasma parameters)

(b) HT restricted to analyzing a single, dominant mode

1. Bandpass filter signal, may be tricky with several modes present or if dominant frequency is varying significantly
2. Amenable to breathing modes, spoke oscillations

Langmuir Probe Measurements of Spoke Oscillations





Parameter	Value	Parameter	Value
P	0.1-10mTorr	n_e	10^{10}-10^{12}cm^{-3}
T_e	4eV	B	10-150G
$\omega_{pi}/2\pi$	2MHz	ρ_g	1mm
λ_D	0.1mm	λ_{n-e}	50cm

- Azimuthal spoke oscillations (~4kHz) occur at low pressure
- A cylindrical, tungsten Langmuir probe of diameter 0.1mm, length 3mm is placed across magnetic field
- LP swept at 50kHz measuring density, plasma potential, and temperature.